Complex numbers in standard form

Recall that the standard form of complex numbers is $a + bi$, where $a, b \in \mathbb{R}$

Exercise 1.2

Prove the following properties for $z, w \in \mathbb{C}$

a. $\mathcal{Re}(z) = \mathcal{Im}(iz)$
b. $\mathcal{Im}(z) = \mathcal{Re}(-iz)$
c. $\bar{z} = 2\mathcal{Re}(z) - z$
d. $|z + w|^2 + |z - w|^2 = 2(|z|^2 + |w|^2)$

Solution Exercise 1.2

let $z = a + bi$, $w = c + di$

a.
$$\begin{aligned} \mathcal{Re}(z) &= \mathcal{Im}(iz) \\ \mathcal{Re}(a + bi) &= \mathcal{Im}(-b + ai) \\ a &= a \end{aligned}$$

b.
$$\begin{aligned} \mathcal{Im}(z) &= \mathcal{Re}(-iz) \\ \mathcal{Im}(a + bi) &= \mathcal{Re}(b - ai) \\ b &= b \end{aligned}$$

c.
$$\begin{aligned} \bar{z} &= 2\mathcal{Re}(z) - z \\ a - bi &= 2a - a - bi \\ a - bi &= a - bi \end{aligned}$$

d.
$$\begin{aligned} |z + w|^2 + |z - w|^2 &= 2(|z|^2 + |w|^2) \\ (a + c)^2 + (b + d)^2 + (a - c)^2 + (b - d)^2 &= 2(a^2 + b^2 + c^2 + d^2) \\ 2(a^2 + c^2 + b^2 + d^2) &= 2(a^2 + b^2 + c^2 + d^2) \end{aligned}$$